Math 240 - Rimmer
Exam # 1  Spring 2017

First and Last Name ____________________________________(PRINT)    Penn ID_________________

Recitation (circle one): W 8       W 9       F 8      F 9
201     202     203         204

W 8       W 9       F 8      F 9
205  206     207        208

Cody

Jay

This exam has 9 questions. Partial credit will be given for the entire exam so be sure to show all work. Give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page, please indicate this in some way.

You have 75 minutes to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5”X11” sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 75 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Once you have completed the exam, sign the academic integrity statement below.
Do NOT write in the grid below. It is for grading purposes only.

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My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

____________________________
Name (printed)

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Signature
1. Solve the system of equations by Gaussian elimination.

\[
\begin{align*}
  x + y + 2z &= -1 \\
  x - 2y + z &= -5 \\
  3x + y + z &= 3
\end{align*}
\]
2. Find a 2 X 2 symmetric matrix that is not invertible

Prove: If A be an $n \times n$ invertible symmetric matrix, then $A^{-1}$ is symmetric.
3. Let $A$ be an $m \times n$ matrix.

   a) Explain why you can always find the product $AA^T$.

   b) What type of special matrix will $AA^T$ be and why?

   c) Let

   $$A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

   Use Math 114 terms about vectors to write out the entries of $AA^T$ in terms of $v =$ Row 1 of $A$ and $w =$ Row 2 of $A$. 
4. Let

\( M \) be the 3\times3 matrix of all 1's.
Let \( A = I_3 - M \), where \( I_3 \) is the 3\times3 identity matrix.

a) Find \( \det(A) \).

b) Find \( N = 2A^{-1} \).
5. Let

\[
A = \begin{pmatrix}
1 & 2 & 0 & 1 \\
2 & 4 & 1 & 4 \\
3 & 6 & 3 & 9
\end{pmatrix}.
\]

a) Find rank \( A \).

b) Find a basis for the Nullspace \( \text{Null}(A) \).

c) Let \( b = \begin{pmatrix} 4 \\ 11 \\ 21 \end{pmatrix} \) and \( x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \). Solve the system \( Ax = b \)
6. Let

\[ S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -4 \\ k \end{pmatrix} \right\} \]. For what value(s) of \( k \) are the vectors linearly dependent?
7. An elementary matrix is one obtained by performing a single operation on the identity matrix.

Find the 3 X 3 elementary matrix for each of the following row operations:

a) Switch Row 1 and Row 3
   
   \[ E_1 = \]

b) Multiply Row 2 by \( \frac{1}{3} \)
   
   \[ E_2 = \]

c) Add -3 Row 1 to Row 2
   
   \[ E_3 = \]

d) Multiply \( M = E_3E_2E_1 \)

\[
M = \begin{bmatrix}
0 & 1 & 2 \\
6 & 4 & 8 \\
1 & 0 & 7
\end{bmatrix}
\]

e) Multiply

\[
\begin{bmatrix}
0 & 1 & 2 \\
6 & 4 & 8 \\
1 & 0 & 7
\end{bmatrix}
\]
8. a) How many 2 X 2 matrices are there that have each entry as either 0 or 1?

b) Does the set of all such matrices form a subspace of the set of 2 X 2 matrices with real entries?

c) Find all such matrices that are invertible.
9. Does the set of polynomials in $P_2$ that satisfy $\int_{-1}^{1} p(x) \, dx = 0$ form a subspace of $P_2$?

If yes, find the standard basis for this subspace.
Scrap Paper
If you use this page and intend for me to look at it, then you must indicate so on the page with the original problem on it. Make sure you label your work with the corresponding problem number.

Do NOT rip this page off.