“My signature below certifies that I have complied with the University of Pennsylvania’s Code of Academic Integrity in completing this”

Signature: ______________________________

Read all of the following information before starting the exam:

• Check your exam to make sure all pages (5 problems plus scratch paper) are present.
• The exam questions are not in a particular order. If you get stuck, move on to the next problem.
• No electronics are allowed; you may use one 8.5x11 sheet (front and back) of handwritten notes.
• You must leave the scratch paper with your exam.
• You MUST show work to receive credit.
• Good luck!

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1.

a. Find the nullity of the adjoined matrix $[A \mid B]$ where $A$ and $B$ are any invertible $n \times n$-matrices. Explain your reasoning.

b. Consider the function $F$ from $\mathbb{R}[x]$ to $\mathbb{R}[x]$ (recall that $\mathbb{R}[x]$ is the collection of all polynomials in variable $x$ with real coefficients) that drops the constant term: for instance, $F(x^2+3x−1) = x^2 + 3x$. Is $F$ a linear transformation? Explain why or why not.

c. T/F: If $A$ is a $3 \times 3$-matrix and $\lambda$ is an eigenvalue of $A$, then $\lambda^3$ is an eigenvalue of $A^3$. If true, explain why; otherwise describe a counterexample.

d. Briefly but specifically explain the purpose of the Wronskian.
2.

a. Find bases of the row space, null space, and column space of the following matrix $A$:

\[
A = \begin{bmatrix}
1 & -2 & 7 & 5 \\
-2 & -1 & -9 & -7 \\
1 & 13 & -8 & -4
\end{bmatrix}
\]

b. Give the general solution of the equation $Ax = b$ where $A$ is the matrix in part (a) and $b$ is the vector $[-5, 5, 10]^T$. 
3. Determine a basis for the subspace of $M_{2 \times 2}(\mathbb{R})$ (remember this is the vector space of $2 \times 2$-matrices with real entries) spanned by:

\[
\begin{bmatrix}
1 & 3 \\
-1 & 2
\end{bmatrix},
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix},
\begin{bmatrix}
-1 & 4 \\
1 & 1
\end{bmatrix},
\begin{bmatrix}
5 & -6 \\
-5 & 1
\end{bmatrix}
\]

Be sure to explain your reasoning.
4. Let $T : P_4(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be the linear transformation defined by $T(p(x)) = p'(x)$ (that is, $T$ takes polynomial $p$ to its derivative $p'$). Relative to the standard bases $B = \{1, x, x^2, x^3, x^4\}$ of $P_4(\mathbb{R})$ and $C = \{1, x, x^2, x^3\}$ of $P_3(\mathbb{R})$, find $[T]_B^C$ and $[T(v)]_C$ for the vector $v = p(x) = 3 - 4x + 6x^2 + 6x^3 - 2x^4$. 
5. Find the eigenvalues and bases for each eigenspace of:

\[
A = \begin{bmatrix}
1 & -2 & 1 \\
-1 & 0 & 1 \\
-1 & -2 & 3
\end{bmatrix}
\]