No books, paper or any electronic device may be used, other than a hand-written note sheet at most $8.5'' \times 11''$ in size. Please turn off your cell phones.

This examination consists of ten long-answer questions, each question is worth 10 points. Please show all your work. Merely displaying some formulas is not sufficient ground for receiving partial credits. Please box your answers.

NAME (PRINTED):

PENN ID:

INSTRUCTOR:

TA:

RECITATION TIME:

My signature below certifies that I have complied with the University of Pennsylvania’s code of academic integrity in completing this examination.

Your signature

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1. True or False. You need to give brief justification for your answer. No credit if you don’t give reasons to support your answer. Let $A$ be an arbitrary $3 \times 2$ matrix

(a) $A^T A$ is always symmetric.
(b) $A^T A$ is always invertible.
2. Let $A$ be a $3 \times 3$ matrix. Suppose we know $Ax = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ has exactly one solution. Based on this information,

(a) What can we say about the number of solutions to the equation $Ax = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$?

(b) What is the rank of $A$?

Justify your answer completely.
3. Which of the followings are vector spaces? Justify your answer completely.

(a) All $2 \times 2$ matrices $N$ such that $N^2 = 0$.
(b) $S = \{(x, y) \in \mathbb{R}^2 \text{ such that } |x| + |y| = 1\}$
(c) All increasing functions over $(-\infty, \infty)$.
(d) All $3 \times 3$ matrices $A$ such that $Ax = \vec{0}$ has infinitely many solutions.
(e) All functions $f$ such that $f(0) = 1$. 
4. Solve the equation \( Ax = b \) using Gaussian elimination, where

\[
A = \begin{bmatrix}
1 & 3 & 5 \\ 
3 & -2 & 11 \\ 
2 & -2 & 6 
\end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}.
\]

Note that the specific elementary row operation used at each step should be given explicitly.
5. Let 

\[ A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}. \]

Is \( A \) invertible? If so find the inverse of \( A \).
6. (a) Compute the determinant of \[
\begin{bmatrix}
0 & 1 & -1 \\
2 & 9 & 8 \\
0 & 0 & 5
\end{bmatrix}
\].

(b) Suppose \[
\begin{vmatrix}
2a & 6b & 2c \\
d & 3e & f \\
g & 3h & i
\end{vmatrix}
\] = 6,

What is \[
\begin{vmatrix}
a & d & g \\
b & e & h \\
c & f & i
\end{vmatrix}
\]?
7. Find the vector space spanned by the column vectors of the matrix.

\[
\begin{bmatrix}
1 & 2 & 0 \\
2 & 0 & 2 \\
2 & 1 & 1 \\
\end{bmatrix}
\]
8. Find all values of $k$ for which the following set is linearly independent.

\[
\{(1, 0, 1, k), (-1, 0, k, 1), (2, 0, 1, 3)\}
\]
9. (a) Compute the Wronskian of \{1, (x + 1)^2, e^x\}.
(b) Is the set \{1, (x + 1)^2, e^x\} linearly dependent or linearly independent?
10. Find the change-of-basis matrix $P_{C \leftarrow B}$ for

$$V = \mathbb{R}^2; B = \{(7, 1), (4, -1)\}; C = \{(2, 1), (-3, 1)\}.$$
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