Math 312: Problem Set 2

Due date: Wed, Sep 16.

1. Find a row-reduced echelon matrix which is row-equivalent to

\[ A = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1 + i \end{pmatrix} \]

What are the solutions of \( AX = 0 \)?

2. Describe explicitly all \( 2 \times 2 \) row-reduced echelon matrices. (Hint: First think about all possible \( r \), the number of non-zero rows.)

3. Consider the system of equations

\[
\begin{align*}
x + y - z &= a \\
x - y + 2z &= b \\
3x + y &= c
\end{align*}
\]

a) Find the general solution of the homogeneous equation.

b) If \( a = 1 \), \( b = 2 \), \( c = 4 \), then a particular solution of the inhomogeneous equation is \( x = 1 \), \( y = 1 \), \( z = 1 \). Find the most general solution of these inhomogeneous equations.

c) If \( a = 1 \), \( b = 2 \), and \( c = 3 \), show these equations have no solution.

4. Show that the system

\[
\begin{align*}
x_1 - 2x_2 + x_3 + 2x_4 &= 1 \\
x_1 + x_2 - x_3 + x_4 &= 2 \\
x_1 + 7x_2 - 5x_3 - x_4 &= 3
\end{align*}
\]

has no solution.

5. a) If \( A := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) and \( B := \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \), compute \( AB \) and \( A^{10} \).

b) Use a), find a \( 2 \times 2 \) matrix \( A \) so that \( A^{10} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \).
6. Let

\[
A = \begin{pmatrix}
1 & 2 & 1 & 0 \\
-1 & 0 & 3 & 5 \\
1 & -2 & 1 & 1
\end{pmatrix}.
\]

Find a row-reduced echelon matrix \( R \) which is row-equivalent to \( A \) and an invertible \( 3 \times 3 \) matrix \( P \), such that \( R = PA \).

7. Let

\[
A = \begin{pmatrix}
1 & -1 & 2 \\
3 & 2 & 4 \\
0 & 1 & -2
\end{pmatrix}.
\]

Use elementary row operations to discover whether it is invertible, and to find the inverse in case it is.