Math 312: Problem Set 4

Due date: Mon, Sep 28. Turn in homework in class.

1. Is the vector \((3, -1, 0, -1)\) in the subspace of \(\mathbb{R}^4\) spanned by the vectors \((2, -1, 3, 2)\), \((-1, 1, 1, -3)\) and \((1, 1, 9, -5)\)?

2. Are the vectors 
   \[\alpha_1 = (1, 1, 2, 4), \alpha_2 = (2, -1, -5, 2), \alpha_3 = (1, -1, -4, 0), \alpha_4 = (2, 1, 1, 6)\]
   linearly independent in \(\mathbb{R}^4\)?

3. Find a basis for the subspace of \(\mathbb{R}^4\) spanned by the four vectors of Problem 2.

4. Show that the vectors 
   \[\alpha_1 = (1, 0, -1), \alpha_2 = (1, 2, 1), \alpha_3 = (0, -3, 2)\]
   form a basis for \(\mathbb{R}^3\).

5. Use \(\alpha_1, \alpha_2, \alpha_3\) from Problem 4. Express each of the standard basis vectors as linear combinations of \(\alpha_1, \alpha_2, \alpha_3\). Use the basis \(\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}\) for the vector space \(\mathbb{R}^3\). What are the coordinates of \((a, b, c)\) with respect to this basis \(\mathcal{B}\)?

6. Fix a real number \(\theta\). Show that the vectors 
   \[\alpha_1 = (\cos \theta, \sin \theta), \alpha_2 = (-\sin \theta, \cos \theta)\]
   form a basis for \(\mathbb{R}^2\). For any vector \((x_1, x_2) \in \mathbb{R}^2\), write down the new coordinates with respect to this new basis \(\mathcal{B} = \{\alpha_1, \alpha_2\}\). (The identity you will need is \(\cos^2 \theta + \sin^2 \theta = 1\).)