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Let $T$ be a linear operator on $\mathbb{R}^3$ represented relative to the standard basis by the matrix

$$
\begin{pmatrix}
1 & -1 & 0 \\
-1 & 1 & 0 \\
0 & 0 & 2
\end{pmatrix}
$$

a) Find all the eigenvalues of $T$

b) Find a basis consisting of eigenvectors of $T$.

c) What is the matrix representing $T$ relative to the basis you found in b)?
Show that the determinant of

$$\begin{pmatrix}
bc & b + c & 1 \\
ca & c + a & 1 \\
ab & a + b & 1
\end{pmatrix}$$

is \((a - b)(b - c)(c - a)\).
Let $A$ be a $3 \times 3$ complex matrix. Suppose the characteristic polynomial for $A$ is $(x - 2i)x^2$.

a) Find all possible minimal polynomials for $A$.

b) Classify such a matrix up to similarity by their Jordan forms.
Let $A$ be the following $3 \times 3$ matrix. Find the Jordan form $J$ for $A$, and the invertible matrix $P$ such that $P^{-1}AP = J$.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$
Let $A$ be the following $4 \times 4$ matrix. Find the Jordan form $J$ for $A$, and the invertible matrix $P$ such that $P^{-1}AP = J$.

$$A = \begin{pmatrix}
7 & 1 & 2 & 2 \\
1 & 4 & -1 & -1 \\
-2 & 1 & 5 & -1 \\
1 & 1 & 2 & 8
\end{pmatrix}$$