

$$= \int_0^1 2\pi x (e^x + \frac{1}{4} e^{-x}) dx$$

$$\textcircled{4} \int 2\pi x (e^x + \frac{1}{4} e^{-x}) dx$$

$$u = 2\pi x$$

$$du = 2\pi dx$$

$$v = e^x - \frac{1}{4} e^{-x}$$

$$dv = e^x + \frac{1}{4} e^{-x} dx$$

$$\text{so } \int 2\pi x (e^x + \frac{1}{4} e^{-x}) dx$$

$$= 2\pi x (e^x - \frac{1}{4} e^{-x}) - \int (e^x - \frac{1}{4} e^{-x}) \cdot 2\pi dx$$

$$= 2\pi x (e^x - \frac{1}{4} e^{-x}) - 2\pi (e^x + \frac{1}{4} e^{-x}) + C$$

$$\textcircled{5} A = \int_0^1 2\pi x (e^x + \frac{1}{4} e^{-x}) dx$$

$$= \left[ 2\pi x (e^x - \frac{1}{4} e^{-x}) - 2\pi (e^x + \frac{1}{4} e^{-x}) \right]_0^1$$

$$= \left[ 2\pi (e - \frac{1}{4} e^{-1}) - 2\pi (e + \frac{1}{4} e^{-1}) \right]$$

$$- \left[ 0 - 2\pi (1 + \frac{1}{4}) \right]$$

$$= -2\pi \frac{1}{4} e^{-1} - 2\pi \frac{1}{4} e^{-1}$$

$$+ 2\pi (1 + \frac{1}{4})$$

$$= \boxed{\frac{5}{2} \pi - \frac{1}{2e} \pi}$$