

5. Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} 3^n \cdot 5^{2-n}$$

If it is convergent, find its sum. (10 points)

$$\begin{aligned} 3^n \cdot 5^{2-n} &= 5^2 \cdot 3^n \cdot 5^{-n} \\ &= 25 \cdot \left(\frac{3}{5}\right)^n \end{aligned}$$

$$\text{so } \sum_{n=1}^{\infty} 3^n \cdot 5^{2-n} = \sum_{n=1}^{\infty} 25 \cdot \left(\frac{3}{5}\right)^n$$

It is an geometric series

$$a = 25, \quad r = \frac{3}{5}, \quad |r| < 1$$

so it is convergent.

The ~~sum~~ series begins with $n=1$ $(S = \frac{ar}{1-r})$

$$\begin{aligned} \text{so } \sum_{n=1}^{\infty} 3^n \cdot 5^{2-n} &= \frac{25 \cdot \frac{3}{5}}{1 - \frac{3}{5}} \\ &= \boxed{\frac{75}{2}} \end{aligned}$$