

8. Do the indefinite integral

$$\int \frac{x^3 + 2x^2 + x + 1}{x^4 + x^2} dx$$

(10 points)

① It is proper

② $x^4 + x^2 = x^2(x^2 + 1)$

③ so
$$\frac{x^3 + 2x^2 + x + 1}{x^4 + x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$$

$$x^3 + 2x^2 + x + 1 = \frac{A}{x} x^2(x^2 + 1) + \frac{B}{x^2} x^2(x^2 + 1) + \frac{(Cx + D)}{x^2 + 1} x^2(x^2 + 1)$$

$$= A x(x^2 + 1) + B(x^2 + 1) + Cx^3 + Dx^2$$

$$= Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$$

$$= (A + C)x^3 + (B + D)x^2 + Ax + B$$

so ~~$A=1, B=$~~ compare the coefficients

$$C + A = 1$$

$$B + D = 2$$

$$B = 1$$

$$A = 1$$

so
$$\begin{cases} A = 1 \\ B = 1 \\ C = 0 \\ D = 1 \end{cases}$$

$$\frac{x^3 + 2x^2 + x + 1}{x^4 + x^2} = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^2 + 1}$$