

7. Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{2n+3}{n!}$$

where $n! = n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1$. (10 points)

① Notice that $\frac{2n+3}{n!} = \frac{2n}{n!} + \frac{3}{n!}$

For $n \geq 2$

$$\begin{aligned} n! &= n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1 \\ &= n \cdot (n-1)! \end{aligned}$$

so for $n \geq 2$

$$\frac{2n}{n!} = \frac{2n}{n \cdot (n-1)!} = \frac{2}{(n-1)!}$$

A boxed diagram showing the simplification of the fraction $\frac{2n}{n!}$. The numerator is $2n$ and the denominator is $n \cdot (n-1)!$. A horizontal line is drawn across the fraction. Below the line, the n in the numerator and the n in the denominator are crossed out, leaving $\frac{2}{(n-1)!}$. The final result is $\frac{2}{(n-1)!} + \frac{3}{n!}$.

so for $n \geq 2$ $\frac{2n+3}{n!} = \frac{2}{(n-1)!} + \frac{3}{n!}$

② We can show that $\sum_{n=2}^{\infty} \frac{2}{(n-1)!}$ and $\sum_{n=2}^{\infty} \frac{3}{n!}$

are both convergent

For $\sum_{n=2}^{\infty} \frac{3}{n!}$, notice for $n \geq 2$

$$n! \geq n(n-1)$$

$$\text{so } 0 \leq \frac{3}{n!} \leq \frac{3}{n(n-1)}$$

and $\sum_{n=2}^{\infty} \frac{3}{n(n-1)}$ is convergent,

so by comparison theorem, $\sum_{n=1}^{\infty} \frac{3}{n!}$ is convergent.