

4. Determine the convergence or divergence of the sequence

$$\left\{ \frac{n \ln n}{n^3 + 3} \right\}$$

If it is convergent, find its limit. (5 points)

Since  $n^3 + 3 > n^3$

$$0 < \frac{1}{n^3 + 3} < \frac{1}{n^3}$$

Since  $n \ln n \geq 0$

we have  $0 \leq \frac{n \ln n}{n^3 + 3} \leq \frac{n \ln n}{n^3} = \frac{\ln n}{n^2}$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n^2} = \lim_{x \rightarrow \infty} \frac{\ln x}{x^2}$$

(L'Hospital's rule)

$$= \lim_{x \rightarrow \infty} \frac{(\ln x)'}{(x^2)'}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{2x} = \lim_{x \rightarrow \infty} \frac{1}{2x^2} = 0$$

By squeeze theorem

$$\boxed{\lim_{n \rightarrow \infty} \frac{n \ln n}{n^3 + 3} = 0}$$

convergent