Mathematics 371 Fall, 2012 (SHATZ)
Assignment #1, due in class October 2, 2012.

We begin with ring theory: Chapters 11, 12.

Part A (do not hand in). (References to Artin)
AI) 354/1.7 b + 1.9
AII) 354/2.2
AIII) 355/3.5
AIV) 355/3.6
AV) 356-7/6.8 a, b, c, d

Part B (to be handed in).

BI) (Perfect medians)
We consider the integers $1, 2, 3, 4, \dots, n$. If r is one of these integers, not n, we say n *has a perfect median*, r, provided that
$$1+2+\cdots+r-1 = (r+1)+(r+2)+\cdots+n.$$
For example, 8 has a perfect median, and it is $r=6$, because
$$1+2+3+4+5 = 7+8 \qquad (=15)$$
Not all integers, n, have perfect medians.
a) Find the *smallest* integer >20 that has a perfect median + find the median. Same for integers >100.
b) Are there infinitely many n that have perfect medians or must the list stop? Why?
c) Give a procedure or formula to find all integers n having perfect medians + find the medians.

BII) Master problem 354/2.2 (AII above) and read pgs 325–330, esp. Prop. 11.3.4(a). We replace the polynomial ring $R[x]$ in that proposition by the formal power series ring $R[[x]]$. We want to extend the homomorphism $\varphi: R \to R'$ to a homomorphism $R[[x]] \to R'$ (as in 11.3.4(a)) sending $x \mapsto \alpha$ with $\alpha \in R'$. But, for this to work we need a condition on α. What is this condition? Using it, prove the analog of 11.3.4(a) for $R[[x]]$.

BIII) a) 355/3.13
b) 355/3.9 (a)+(b).

BIV) Let F be a field (E.g. $F = \mathbb{Q}, \mathbb{R}, \mathbb{C}$, or any other field you know, your choice). Write $\mathcal{M}_{p,q}(F)$ for the collection of all $p \times q$ matrices with entries in F. Choose a $\underline{q \times p}$ matrix, Γ, and

make $\mathcal{M}_{pq}(F)$ a ring via: Addition as usual among $p\times q$ matrices, Multiplication: If $R, S \in \mathcal{M}_{pq}(F)$ set $R * S = R\Gamma S$ (ordinary product of matrices on right hand side). Write $\mathcal{M}(\Gamma)$ for this new ring; it depends on Γ (we hold $p + q$ fixed).
a) Prove that up to isomorphism there are only finitely many rings $\mathcal{M}(\Gamma)$.
b) Show that if there is $Q \in \mathcal{M}_{pq}(F)$ with $\Gamma Q \Gamma = \Gamma$, then by multiplying Γ on the left and right by W, Z (to get $W\Gamma Z$) with W $q\times q$ and Z $p \times p$, we can get