Math 114 Quiz & HW of Week 12 Selected Solutions

Dec 4, 2009

Quiz 8(16.3, #23 16.4 #17)

Monday Nov 30

1. Find the volume of the solid bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

$V = \int_0^2 \int_0^{3-3x/2} (6 - 3x - 2y)dydx$

$= \int_0^2 [6y - 3xy - y^2]_{y=0}^{y=3-3x/2} dx$

$= \int_0^2 (9x^2/4 - 9x + 9)dx = 6$

2. Find the area of the region within both circles $r = \cos\theta$ and $r = \sin\theta$.

(To draw the two circles you can convert them into rectangular coordinates.)

By symmetry,

$A = 2\int_0^{\pi/4} \int_0^{\sin\theta} r dr d\theta = (\pi - 2)/8.$

Wednesday Dec 2

1. Find the volume of the solid bounded by the coordinate planes and the plane $2x + 3y + z = 6$.

$V = \int_0^2 \int_0^{3-3y/2} (6 - 3y - 2x)dxdy$

$= \int_0^2 [6x - 3yx - x^2]_{x=0}^{x=3-3y/2} dy$

$= \int_0^2 (9y^2/4 - 9y + 9)dy = 6$

2. Find the area of the region within both circles $r = \cos\theta$ and $r = \sin\theta$.

(To draw the two circles you can convert them into rectangular coordinates.)

By symmetry,

$A = 2\int_0^{\pi/4} \int_0^{\sin\theta} r dr d\theta = (\pi - 2)/8.$

HW11:

I graded 16.5 #14, 16.6 #22 for correctness and others for completion.

16.5

12. $\rho(x, y) = k(x^2 + y^2) = kr^2$, $m = \int_0^{\pi/2} \int_0^1 kr^3 dr d\theta = \frac{\pi}{8}k$,

$M_y = \int_0^{\pi/2} \int_0^1 kr^4 \cos\theta dr d\theta = \frac{1}{5}k$, $M_x = \int_0^{\pi/2} \int_0^1 kr^4 \sin\theta dr d\theta = \frac{1}{5}k$.

So $(\bar{x}, \bar{y}) = (\frac{8}{5\pi}, \frac{8}{5\pi})$.

16. $(\bar{x}, \bar{y}) = (0, \frac{2\sqrt{3}}{2(3\sqrt{3}-\pi)})$.

28. (a) $f(x, y) \geq 0$, so it suffices to check $\iint_{\mathbf{R}^2} f(x, y)dA = 1$.

(b) (i) $P(X \geq 1/2) = \int_{1/2}^{\infty} \int_{-\infty}^{\infty} f(x, y)dydx = 3/4$

(ii)$P(X \geq 1/2, Y \leq 1/2) = 3/16$.

(c) The expected value of X is given by

$$\mu_1 = \iint_{\mathbf{R}^2} xf(x,y)dA = 2/3$$

The expected value of Y is given by

$$\mu_1 = \iint_{\mathbf{R}^2} yf(x,y)dA = 2/3$$

30(b) The lifetime of each bulb has exponential dencity function

$$f(t) = \begin{cases} 0 & \text{if t} < 0 \\ \frac{1}{1000}e^{-t/1000} & \text{if t} \geq 0 \end{cases}$$

So we want to find $P(X+Y \leq 1000)$, or equivalently $P((X,Y) \in D)$, where D is the triangular region bounded by $x+y=1000$ and the coordinate axes.

So $P(X+Y \leq 1000) = \iint_D f(x,y)dA =$
$\int_0^{1000} \int_0^{1000-x} 10^{-6}e^{-(x+y)/1000}dydx = 1 - 2e^{-1} \approx 0.2642$

16.6

14. 3/28

22. The solid is $E = \{(x,y,z)|y^2+z^2 \leq x \leq 16\}$, if we let $D = \{(y,z)|y^2+z^2 \leq 16\}$ and use polar coordinates $y = r\cos\theta$, $z = r\sin\theta$
$V = \iint_D (\int_{y^2+z^2}^{16} dx)dA = \int_0^{2\pi} \int_0^4 (\int_{y^2+z^2}^{16} dx)rdrd\theta = 128\pi$

16.7

8. Since $2r^2+z^2 = 1$ and $r^2 = x^2+y^2$, we have $2(x^2+y^2)+z^2 = 1$, and ellipsoid centered at the origin with intercepts $x = \pm 1/\sqrt{2}$, $y = \pm 1/\sqrt{2}$, $z = \pm 1$.

18. 2/35

22. $\frac{4}{3}\pi(8-3^{3/2})$.