Quiz 8 (16.3, #23 16.4 #17)

Monday Nov 30
1. Find the volume of the solid bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

$$V = \int_0^2 \int_{3x/2}^3 (6 - 3x - 2y) \, dy \, dx$$
$$= \int_0^2 (6y - 3xy - y^2)_{y=3}^{y=3x/2} \, dx$$
$$= \int_0^2 (9x^2/4 - 9x + 9) \, dx = 6$$

2. Find the area of the region within both circles $r = \cos \theta$ and $r = \sin \theta$.

(To draw the two circles you can convert them into rectangular coordinates.)

By symmetry,

$$A = 2 \int_0^{\pi/4} \int_0^{\sin \theta} r \, dr \, d\theta = (\pi - 2)/8.$$ 

Wednesday Dec 2
1. Find the volume of the solid bounded by the coordinate planes and the plane $2x + 3y + z = 6$.

$$V = \int_0^2 \int_{3y/2}^3 (6 - 3y - 2x) \, dx \, dy$$
$$= \int_0^2 (6x - 3yx - x^2)_{x=0}^{x=3y/2} \, dy$$
$$= \int_0^2 (9y^2/4 - 9y + 9) \, dy = 6$$

2. Find the area of the region within both circles $r = \cos \theta$ and $r = \sin \theta$.

(To draw the two circles you can convert them into rectangular coordinates.)

By symmetry,

$$A = 2 \int_0^{\pi/4} \int_0^{\sin \theta} r \, dr \, d\theta = (\pi - 2)/8.$$ 

HW11:

I graded 16.5 #14, 16.6 #22 for correctness and others for completion.

16.5

12. \(\rho(x, y) = k(x^2 + y^2) = kr^2\), \(m = \int_0^{\pi/2} \int_0^1 kr^3 d\theta \, dr = \frac{\pi}{8} k\),

\(M_y = \int_0^{\pi/2} \int_0^1 kr^4 \cos \theta \, dr \, d\theta = \frac{1}{5} k\), \(M_x = \int_0^{\pi/2} \int_0^1 kr^4 \sin \theta \, dr \, d\theta = \frac{1}{5} k\).

So \((\bar{x}, \bar{y}) = \left(\frac{8}{5\pi}, \frac{8}{5\pi}\right)\).

16. \((\bar{x}, \bar{y}) = \left(0, \frac{2\sqrt{3}}{2(3\sqrt{3} - \pi)}\right)\).

28. (a) \(f(x, y) \geq 0\), so it suffices to check \(\iint_{R^2} f(x, y) \, dA = 1\).

(b) (i) \(P(X \geq 1/2) = \int_{1/2}^\infty \int_{-\infty}^\infty f(x, y) \, dy \, dx = 3/4\)

(ii)\(P(X \geq 1/2, Y \leq 1/2) = 3/16\).
(c) The expected value of $X$ is given by
\[ \mu_1 = \iint_{\mathbb{R}^2} x f(x, y) dA = \frac{2}{3} \]
The expected value of $Y$ is given by
\[ \mu_1 = \iint_{\mathbb{R}^2} y f(x, y) dA = \frac{2}{3} \]
30(b) The lifetime of each bulb has exponential density function
\[ f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{1000} e^{-t/1000} & \text{if } t \geq 0 \end{cases} \]
So we want to find $P(X + Y \leq 1000)$, or equivalently $P((X, Y) \in D)$, where $D$ is the triangular region bounded by $x + y = 1000$ and the coordinate axes.
So
\[ P(X + Y \leq 1000) = \iint_D f(x, y) dA = \int_0^{1000} \int_0^{1000-x} 10^{-6} e^{-(x+y)/1000} dydx = 1 - 2e^{-1} \approx 0.2642 \]
16.6
14. 3/28
22. The solid is $E = \{(x, y, z)|y^2 + z^2 \leq x \leq 16\}$, if we let $D = \{(y, z)|y^2 + z^2 \leq 16\}$ and use polar coordinates $y = r \cos \theta$, $z = r \sin \theta$
\[ V = \iint_D (\int_{y^2+z^2}^{16} dx) dA = \int_0^{2\pi} \int_0^4 (\int_{y^2+z^2}^{16} dx) r dr d\theta = 128\pi \]
16.7
8. Since $2r^2 + z^2 = 1$ and $r^2 = x^2 + y^2$, we have $2(x^2 + y^2) + z^2 = 1$, and ellipsoid centered at the origin with intercepts $x = \pm 1/\sqrt{2}$, $y = \pm 1/\sqrt{2}$, $z = \pm 1$.
18. 2/35
22. $\frac{4}{3} \pi (8 - 3^{3/2})$. 