Math 114 Quiz & HW of Week 7 Selected Solutions

Nov 1, 2009

Quiz 5

1. Find the length of the curve:

\[ r(t) = \sqrt{2}i + e^t j + e^{-t}k, \quad 0 \leq t \leq 1. \]

\[ r'(t) = \sqrt{2}i + e^t j - e^{-t}k \Rightarrow |r'(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}. \]

Hence \( L = \int_0^1 |r'(t)| dt = \int_0^1 (e^t + e^{-t}) dt = e - e^{-1}. \)

2. Find the tangential component of the acceleration vector:

\[ r(t) = (3t - t^3)i + 3t^2j. \]

\[ r(t) = (3t - t^3)i + 3t^2j \Rightarrow r'(t) = (3 - 3t^2)i + 6tj, \]

\[ |r'(t)| = \sqrt{(3 - 3t^2)^2 + (6t)^2} = 3 + 3t^2, \]

\[ r''(t) = -6ti + 6j, \quad r'(t) \times r''(t) = (18 + 18t^2)k. \]

Then Equation 9 gives \( a_T = 6t \) (Equation 8 also works.)

HW7:

I graded 15.2 #16, 15.3 #36 for correctness and others for completion.

week 7

15.2

10. Does not exist.

16. We can use the Squeeze Theorem to show that the limit is 0:

\[ 0 \leq \frac{x^2 + \sin^2 y}{x^2 + y^2} \leq \sin^2 y, \] then take the limit. Another way is, by Ethan Aaron, using polar coordinates. Letting \( r \to 0^+ \), we also get 0.

34. \( G(x, y) = g(f(x, y)) \) where \( f(x, y) = (x + y)^2 \), a rational function that is continuous on \( \mathbb{R}^2 \) except where \( x + y = 0 \), and \( g(t) = \tan^{-1} t \), continuous everywhere. Thus \( G \) is continuous on its domain \{ \( (x, y) \mid x + y \neq 0 \} \).

38. By letting \( x = 0 \) and \( x = y \) respectively, you can see that \( f \) is not continuous at \( (0, 0) \).

40. 0

15.3

22. \( f(x, y) = x^y \Rightarrow f_x(x, y) = yx^{y-1}, f_y(x, y) = x^y \ln x. \)

28. Use the fundamental Theorem of Calculus.

\[ f_x(x, y) = \frac{\partial}{\partial x} \int_y^x \cos(t^2) dt = \cos(x^2), \quad f_y(x, y) = -\cos(y^2). \]

36. \( f_x(x, y, z, t) = \frac{y^2}{t + 2z}, \quad f_y(x, y, z, t) = \frac{2xy}{(t + 2z)^2}, \quad f_z(x, y, z, t) = -\frac{2xy^2}{(t + 2z)^3}, \quad f_t(x, y, z, t) = -\frac{xy^2}{(t + 2z)^2} \)