

**(1) Common Initials.**

- (a) How many people need to be members of a group before we can be certain that two people have the same first and last initials in English?
- (b) What if we require that they have the same first, middle and last initials? Some people may have no middle name, in which case we count “Blank” as a permitted middle initial.

**(2) Types of Fruit.** Suppose a bag contains unlimited numbers of 1) apples, 2) bananas, 3) oranges, and 4) strawberries.

- (a) How many fruit must you draw at random from the bag before you know that you have 4 fruit of the same type?
- (b) Suppose you drew 5 apples, 2 bananas, 4 oranges, and 2 strawberries. In how many different orders could you have picked those fruit?
- (c) Suppose I want 4 of the same type OR 4 of all different types (i.e. at least one of each type) How many fruit do I need to pick?

**(3) Euler’s phi function.** Recall that  $\varphi(n)$  = the number of integers  $m$  such that  $1 \leq m \leq n$  and the greatest common divisor of  $m$  and  $n$  is 1.

- (a)  $\varphi(120)$
- (b)  $\varphi(p)$  for  $p$  prime.
- (c)  $\varphi(2^n)$  for  $n$  an integer.
- (d)  $\varphi(10^n)$  for  $n$  an integer.